A Compromise Solution for the Fully Fuzzy Multiobjective Linear Programming Problems

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Thursday, May 9, 2019: 2^{nd} ***Seminar***

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ABSTRACT

A new approach is undertaken to solve the fully fuzzy multiobjective linear programming (FFMLP) problem. The coefficients of the objective functions, constraints, right-hand-side parameters, and variables are of the triangular fuzzy number (TrFN)s. A solution strategy, called compromise solution algorithm (CSA), is presented using a three-step procedure.

ABSTRACT

First, a revised simplex method together with Gaussian elimination in the environment of the linear ranking function is used to convert the FFMLP problems partially into semi fully fuzzy multiobjective linear programming (SFFMLP) problems. Then, the obtained SFFMLP problems are gathered together as a single problem. Finally, the gathered problem is solved by one of four different methods to find a fuzzy compromise solution for the FFMLP problems. The CSA is then numerically applied to a FFMLP problem to illustrate the practicability of the proposed procedure.

KEYWORDS

FFMLP problems, SFFMLP problems, FFLP problem, SFFLP problem, CSA, simplex method and Gaussian elimination, fuzzy simplex method, linear ranking function, and fuzzy compromise solution.

INTRODUCTION

Many researchers had been involved in the field of FFMLP problems(1-50).

MOTIVATION OF THE STUDY

the motivation of the study for the proposed research methodology by this illustrative example is for the following reasons:

MOTIVATION OF THE STUDY

- Scientific attention need to be given to the FFMLP problems.
- Finding the optimal solution to the FFLP problem without converting it into its equivalent deterministic is still an open research problem.
- To find a compromise solution for the the FFMLP problems within a range of the proposed research methodologies.

CONCEPTS OF FUZZY NUMBERS, AND RANKING FUNCTIONS

- **The Trapezoidal Fuzzy Number (T_pFN):** Let $\tilde{A} = (a^L, a^U, \alpha, \beta)$ be the T_pFN, where $[a^L, a^U]$ is the modal set of $\tilde{A}$, and $[a^L - \alpha, a^U + \beta]$ its support part (13-15).
- **The Triangular Fuzzy Number (T_rFN):** If $a = a^L = a^U \in \tilde{A}$ then the T_pFN is reduced to T_rFN, and denoted by $\tilde{A} = (a, \alpha, \beta)$.

- **Linear Ranking Function** is a map which transforms each fuzzy number into its corresponding real line, where a natural order exists, mathematically, $\mathfrak{R} : \tilde{A} \rightarrow \mathbb{R}; \forall \tilde{A}$ and $\mathbb{R}$ is the set of all real numbers (41, 45).

FFMLP FORMULATION

$$
\begin{aligned}
&\text{Max. } \tilde{Z}_i(\tilde{x}) \underset{\mathfrak{R}}{=} \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_j \;; i = 1, \ldots, r \\
&\text{Min. } \tilde{Z}_i(\tilde{x}) \underset{\mathfrak{R}}{=} \sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_j \;; i = r+1, \ldots, s \\
&\text{s.t. } \sum_{j=1}^{n} \tilde{a}_{ij}\tilde{x}_j \begin{bmatrix} \leq \\ \geq \\ = \end{bmatrix} \tilde{b}_i \;, i = 1, 2, \ldots, m,
\end{aligned} \tag{1}
$$

FFMLP FORMULATION

where $\tilde{a}_{ij} = (a_{ij}^L, a_{ij}^U, \alpha_{ij}, \beta_{ij}), \tilde{b}_i = (b_i^L, b_i^U, \alpha_i, \beta_i),\ \tilde{c}_j = (c_j^L, c_j^U, \omega_j, \eta_j)$ and $\tilde{x}_j = (x_j^L, x_j^U, \alpha_j, \beta_j)$ are in the set of all $\mathrm{T_pFN}s$, $i = 1, \ldots, m,\ j = 1, \ldots, n$.

FFMLP FORMULATION

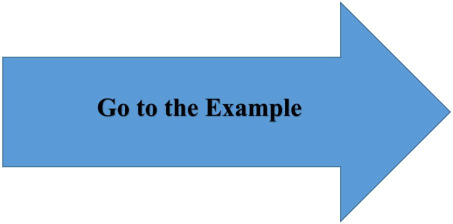
To solve (1), we have to find a set of basic feasible solution $\tilde{x} = \left\{ \tilde{x}_1, \tilde{x}_2, ..., \tilde{x}_{n+m} \right\}$ of fuzzy variables which satisfies the set of all constraints, non-negative restrictions and optimizes (maximizes or minimizes) the objective functions in (1).

PARTIALLY CONVERTING THE FFLP PROBLEM INTO IT EQUIVALENT SFFLP AND SOLUTION ALGORITHM O THE FFMLP PROBLEMS

The following procedure is used to solve FFMLP problem through the linear ranking function as follows:
Consider the FFMLP problem in (1) as

APPLICATION

We present an example of the implementation of the research methodology in the problem statement within the framework of the MFFLP problem.
Solve the following MFFLP problem:


Go to the Example

CONCLUSION

In this paper, the fully fuzzy multiobjective linear programming (FFMLP) problems have been defined, where the coefficients of the objective functions, constraints, right hand side parameters, and variables are of the type of the Triangular Fuzzy Number (T_rFN)s. A solution strategy of such FFMLP was presented in the circumstance of certain linear ranking function, namely, Compromise Solution Algorithm (CSA).

CONCLUSION

The revised simplex method together with Gaussian elimination in the environment of the linear ranking function which was described in the proposed was first converted from FFMLP problem to partially SFFMLP problem. The obtained SFFMLP problems were gathered together, and then was solved by four different methods to find a fuzzy compromise solution. The results show that the proposed CSA is applicable and practicable within computational applications. We intend to expand this research further to practical contributions.

CONCLUSION

The work will be able to help solve real life and industrial problems which are usually complicated, uncertain and continuously subject to changes, by considering the fuzziness in the formulation of the model. In addition, potential research will be the application of our proposed solution procedure to real life problems which usually involve many variables and require quick optimum solutions. We believe that this study will spur other research so as to have positive impacts on organization productivity and competitiveness in many industries.

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